

Elementary particles and fundamental forces

- Upper secondary school -

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Exercise 1. *Standard Model*

According to the current theories of particle physics we consider the following particles to be elementary: the quarks, leptons and interaction bosons. In table 1 one can see listed all the particles with their properties and their names. For every quark and lepton in the table there is a partner particle called anti-particle which has the same properties but with the electric charge reversed. (For instance the up quark is denoted by u and its antiparticle by $\bar{u}$).

In the following we will use the name of quark both to designate the quarks and the anti-quarks. Quarks can participate in combinations of two or three to create composed particles obeying the rule that the total charge to be an entire number. The "top" quark is an exception and doesn't participate in any combinations since it has a very short lifetime. The combinations of two quarks are known as *mesons* and the combinations of three quarks are called *baryons*. All particles made of quarks are also called *hadrons*.

With these information, answer the following questions.

	mass → $\approx 2.3 \text{ MeV}/c^2$ charge → $2/3$ spin → $1/2$ u up	mass → $\approx 1.275 \text{ GeV}/c^2$ charge → $2/3$ spin → $1/2$ c charm	mass → $\approx 173.07 \text{ GeV}/c^2$ charge → $2/3$ spin → $1/2$ t top	mass → 0 charge → 0 spin → 1 g gluon	mass → $\approx 126 \text{ GeV}/c^2$ charge → 0 spin → 0 H Higgs boson
QUARKS	mass → $\approx 4.8 \text{ MeV}/c^2$ charge → $-1/3$ spin → $1/2$ d down	mass → $\approx 95 \text{ MeV}/c^2$ charge → $-1/3$ spin → $1/2$ s strange	mass → $\approx 4.18 \text{ GeV}/c^2$ charge → $-1/3$ spin → $1/2$ b bottom	mass → 0 charge → 0 spin → 1 γ photon	
	mass → $0.511 \text{ MeV}/c^2$ charge → -1 spin → $1/2$ e electron	mass → $105.7 \text{ MeV}/c^2$ charge → -1 spin → $1/2$ μ muon	mass → $1.777 \text{ GeV}/c^2$ charge → -1 spin → $1/2$ τ tau	mass → $91.2 \text{ GeV}/c^2$ charge → 0 spin → 1 Z Z boson	
LEPTONS	mass → $< 2.2 \text{ eV}/c^2$ charge → 0 spin → $1/2$ ν_e electron neutrino	mass → $< 0.17 \text{ MeV}/c^2$ charge → 0 spin → $1/2$ ν_μ muon neutrino	mass → $< 15.5 \text{ MeV}/c^2$ charge → 0 spin → $1/2$ ν_τ tau neutrino	mass → $80.4 \text{ GeV}/c^2$ charge → ± 1 spin → 1 W W boson	GAUGE BOSONS

Figure 1: The Standard Model table of elementary particles. There are three types of particles: **quarks** și **leptons**, **interaction bosons**

1. Compute the total electrical charge of the following combinations of quarks.

a. uud - proton,

b. udd - neutron,

c. $u\bar{d}$ - π meson ,

d. $s\bar{d}$ - K meson ,

e. $c\bar{d}$ - D meson

2. Fill out the composition and the charge of the anti-particle of the example from above

a. anti-proton

_____, _____

b. anti-neutron

_____, _____

c. anti-meson π^+

_____, _____

d. anti-meson K^+

_____, _____

e. anti-meson D^0

_____, _____

3. Compute the total number of combinations that can be formed from two quarks and select the right answer from below.

a. 12

b. 100

c. 36

d. 25

e. 84

4. Compute the total number of combinations that can be formed from three quarks and select the right answer from below.

a. 21

b. 62

c. 125

d. 250

e. 168

5. Which elementary particle is the most abundant in the human organism? Start from the chemical composition of the human body and write down in descending order the three most abundant elementary particles.

a) _____

b) _____

c) _____

Solution of the exercise 1.

1. The combinations of quarks are:

- | | | |
|--|---------------------------------------|--|
| a. uud - proton,
<u>1</u> | b. udd - neutron,
<u>0</u> | c. $u\bar{d}$ - π meson,
<u>1</u> |
| d. $s\bar{d}$ - K meson,
<u>1</u> | e. $c\bar{d}$ - D meson
<u>0</u> | |

2. The composition and the charge of the antiparticles are:

- | | | |
|--|--|--|
| a. anti-proton
<u>$\bar{u}\bar{u}\bar{d}$</u> , <u>-1</u> | b. anti-neutron
<u>$\bar{u}\bar{d}\bar{d}$</u> , <u>0</u> | c. anti-meson π^+
<u>$\bar{u}\bar{d}$</u> , <u>-1</u> |
| d. anti-meson K^+
<u>$\bar{u}d$</u> , <u>-1</u> | e. anti-meson D^0
<u>$\bar{c}d$</u> , <u>0</u> | |

3. Mesons are made of two quarks. To obey the conditions that the charge to an entire number there are possible only combinations of a quark and an antiquark. More specifically, we use letter u for the quarks in the first row of the table of the Standard Model (i.e the quarks "up", "charm" and "top") and with the letter d the quarks on the second row of the table (quarks "down", "strange", "bottom"). The possible quarks combinations are:

- $u\bar{d}$, charge $+1$, number of combinations: 6
- $u\bar{u}$, charge 0, number of combinations: 4
- $d\bar{d}$, charge 0, number of combinations: 9
- $\bar{u}d$, charge -1 , number of combinations: 6

The total number of mesons is 25, the correct answer is **d**) (we excluded the combinations for the "top" quark).

4. Baryons are particles made of three quarks. To have a total charge an entire number the following classes of combinations are possible: uud , udd , ddd and their respective antiparticle combinations $\bar{u}\bar{u}\bar{d}$, $\bar{u}\bar{d}\bar{d}$, $\bar{d}\bar{d}\bar{d}$.

The answer to the question is obtained by summing the number of the combinations in each class. Thus we have:

- uud , charge $+1$, number of combinations: 9
- udd , charge 0, number of combinations: 12
- ddd , charge -1 , number of combinations: 10

The anti-particles are in equal numbers. As an exercise one can write down all the possible combinations, just for illustration we write here the particles from the first class udd .

The combinations are: udd , uds , udb , uss , usb , ubb , cdd , cds , cdb , css , csb , cbb .

The total number of combinations is $2 * (9 + 12 + 10) = 62$, the correct answer is **b**).

5. In nature and in the human body we mostly find up and down quarks and electrons. Other elementary particles are present only occasionally and in tiny quantities. To compute the abundance of each of the three particles we need the chemical composition of the body.

According to sources available on internet, the body is formed by 63% of hydrogen atoms, 18% carbon atoms, 10% oxygen atoms, 3% nitrogen atoms, 1,5% calcium etc. All atoms have an electronic cloud and a nucleus made of protons and neutrons. The quark composition of the proton is uud and for the neutron is udd .

From all three particles the electron is the less abundant since the number of electrons is equal to the number of protons and protons are made of more quarks. The chemical elements O , N , Ca have the same number of protons and neutrons thus the same number of u and d quarks. The only element which makes the distinction is the hydrogen, made only of a proton, two quarks u and one quark d .

The most abundant elementary particles in the human body in decreasing order are:

- a) _____ up quark
- b) _____ down quark
- c) _____ electron

Exercise 2. *The scale of elementary particles and of LHC*

1. If the nucleus of an atom would measure 1 cm, what would be the total dimension of the atom?
 - a. as a sheet of paper
 - b. as a football stadium
 - c. as large as the Bucharest city
 - d. as five football stadiums
 - e. as large as the whole universe
2. What is the total distance travelled by the protons within LHC ring in 10 hours? Assume that the speed of the protons is equal to the speed of light. Which of the following distances is closer to the value obtained?
 - a. The distance between Earth and Sun back and forth
 - b. The distance between Earth and Neptun back and forth
 - c. The distance between London and Paris
 - d. The distance around the Earth
3. At LHC the protons are accelerated in bunches of protons, the average density of a bunch is 10^{11} protons. All the bunches that are in the LHC at a given time form a beam, a beam has around 2800 bunches. When the experiment is finished the beam is extracted from the ring and dumped in a block of concrete foreseen for this purpose. Calculate the energy of the beam given that the energy of a proton is 7 TeV, i.e. 7×10^{12} eV. The electron-volt (eV) is unit used to express the energy of the particles it is equivalent to $1 \text{ eV} = 1,602 \times 10^{-19}$ Joules.
Compare this energy with the energies from the following examples and pick the one which is closer to the one calculated.

- a. a speeding TGV train
- b. a fly
- c. an Airbus plane
- d. a bullet fired by a gun

Indication: for each of the variants consider the typical values for the masses and speeds.

Solution of the exercise 2.

- The dimension of an atomic nucleus is about few femtometers, while for the atom is about the order of angstroms. The ratio of the two is approximately 50000.

For the following calculation we will consider for each example the length.

- a. A sheet of paper $\sim 25 \text{ cm}$
- b. A football stadium $\sim 100 \text{ m} = 10000 \text{ cm}$
- c. Bucharest city $\sim 25 \text{ km} = 25 * 10^5 \text{ cm}$
- d. Five football stadiums $\sim 50000 \text{ cm}$

The correct answer is correct **d**).

- The total distance travelled by the protons in 10 hours is:

$$d = c * t = 300000 \text{ km/s} * 10 * 3600 \text{ s} = 1.08 * 10^{10} \text{ km}$$

- (a) The distance between Earth and Sun back and forth $= 3 * 10^8$
- (b) The distance between Earth and Neptun back and forth $\sim 0.87 * 10^{10} \text{ km}$
- (c) The distance between London and Paris $= 344 \text{ km}$
- (d) The distance around the Earth $= 4 * 10^5 \text{ km}$

The answer **b**) is closest to the computed value.

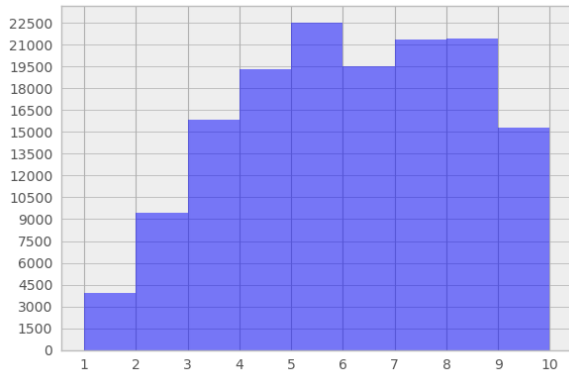
- The total energy of the protons beam is sum of the energy of all the protons from all bunches. That is,

$$E_{total} = N_{bunch} * n_{protoni} * E_{proton} = 2800 * 10^{11} * 7 * 10^{12} * 1.672 * 10^{-19} \text{ J} = 314 \text{ MJ}$$

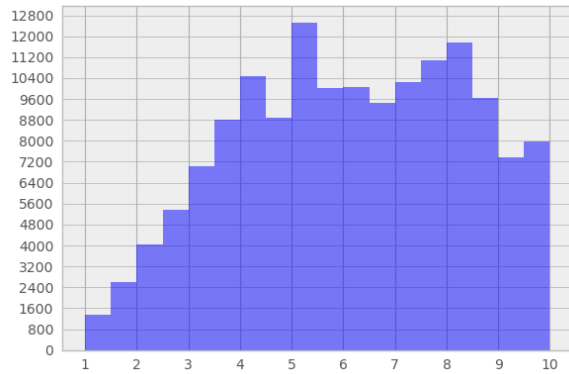
For the next examples we will use the formula $E = \frac{1}{2} m * v^2$.

- a. TGV train $= \frac{1}{2} * 4 * 10^5 \text{ kg} * (200 \text{ km/h})^2 = 617 \text{ MJ}$
- b. fly $= \frac{1}{2} * 0,012 \text{ kg} * (2 \text{ m/s})^2 = 0,024 \text{ J}$
- c. Airbus plane $= \frac{1}{2} * 7 * 10^4 \text{ kg} * (700 \text{ km/h})^2 = 1323 \text{ MJ}$
- d. bullet $= \frac{1}{2} * 0,02 \text{ kg} * (1000 \text{ m/s})^2 = 100 \text{ kJ}$

The closest answer is the answer **a**). The beam of protons has the energy of train of 400 de tones travelling with 140 km/h .



(a) The marks are represented in bins of unit 1.



(b) The marks are represented in bins of unit 0.5.

Figure 2: Histogram plots for the math exams in 2016 in Romania.

Exercise 3. What is a histogram?

A histogram is a common type of plots in the experimental physics. On OX axis we put the values for the quantity of interest and on OY we note the number of times a certain value has appeared after repeated measurements.

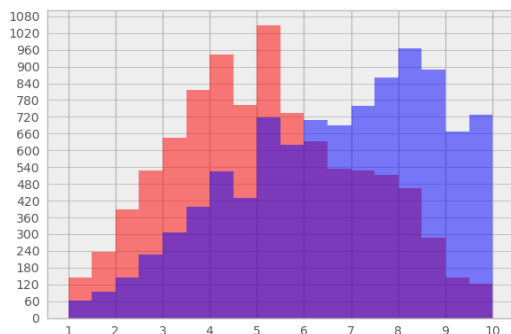
To get accustomed with this quantity we will plot on a histogram the marks from the exams at mathematics for the last grade of the lower secondary school. The data is for the national exam in year 2016 in Romania. On the OX axis are the marks from 0 to 10, 10 is the best mark. On the OY axis we note how many students had a certain mark. The histogram can be seen in the figure 2a. A bin is the interval on the OX axis for which we assign a single value on the OY axis. In the histogram 2a we have 10 bins of dimension 1. Examples of bins are $(0, 1), \dots (4, 5), (5, 6), \dots (9, 10)$.

1. Using the information from the figure 2a, extract the following quantities:

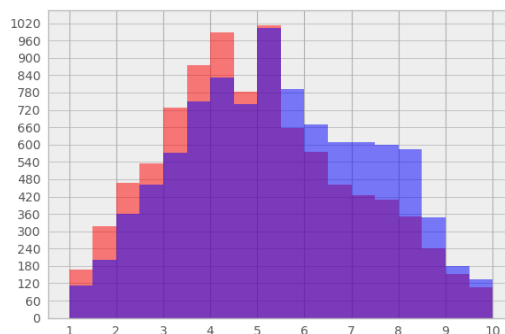
- | | | |
|--|-------------------------------|-------------------------------------|
| a. the number of students which gave the exam, | b. average mark, | c. the most frequent mark assigned, |
| _____ | _____ | _____ |
| d. the lowest mark assigned, | e. the highest mark assigned, | |
| _____ | _____ | |

2. The same information about the marks can be represented in a plot with smaller bins. For instance, the same data is presented in figure 2b as a histogram with bins of 0.5. For this case extract the same quantities as before.

- | | | |
|--|-------------------------------|-------------------------------------|
| a. the number of students which gave the exam, | b. the average mark, | c. the most frequent mark assigned, |
| _____ | _____ | _____ |
| d. the lowest mark assigned, | e. the highest mark assigned, | |
| _____ | _____ | |



(a) The marks in the urban areas (blue) and in the rural areas (red).



(b) The marks of the girls with blue and of the boys with red.

Figure 3: Comparison of the marks for math exam of different groups of students.

Observations: the number of students giving the exam is the same in both cases, the average mark is different being higher in the second case, why?

3. Compare the histograms from the following examples¹

- In figure 3a we have plotted two distributions of marks from the math exam a) of the students from the urban areas (blue) b) of the students from the rural areas (red)². How do the histograms differ, comment the results.
- In figure 3b we have plotted two distributions of marks from the math exam a) of the girls (blue) b) of the boys (red). How do the histograms differ, comment the results.

Solution of the exercise 3.

- Read on the plot 2a the number of students with the mark in a given bin. In this case the values are³:

$[1, 2) : 3942, [2, 3) : 9419, [3, 4) : 15826, [4, 5) : 19338, [5, 6) : 22543, [6, 7) : 19524, [7, 8) : 21333,$
 $[8, 9) : 21410, [9, 10) : 15313$

- | | | |
|--|-------------------------------|-------------------------------------|
| a. the number of students which gave the exam, | b. the average mark, | c. the most frequent mark assigned, |
| <u>148648</u> | <u>5.62</u> | <u>5</u> |
| d. the lowest mark assigned, | e. the highest mark assigned, | |
| <u>1</u> | <u>10</u> | |

¹For making things easier we considered the same number of students in each category.

³Due to the picture resolution each student will read different values. To be more accurate one can average all the reads.

2. Read on the plot [2b](#) the number of students with the mark in a given bin. In this case the values are:

{[1, 1.5) : 1360, [1.5, 2) : 2582, [2, 2.5) : 4051, [2.5, 3) : 5368, [3, 3.5) : 7014, [3.5, 4) : 8812, [4, 4.5) : 10468,
[4.5, 5) : 8870, [5, 5.5) : 12536, [5.5, 6) : 10007, [6, 6.5) : 10057, [6.5, 7) : 9467, [7, 7.5) : 10259,
[7.5, 8) : 11074, [8, 8.5) : 11765, [8.5, 9) : 9645, [9, 9.5) : 7356, [9.5, 10) : 7957}

- | | | |
|---|--|---|
| a. the number of students which gave the exam,
<u>148648</u> | b. the average mark,
<u>5,87</u> | c. the most frequent mark assigned,
<u>5</u> |
| d. the lowest mark assigned,
<u>1</u> | e. the highest mark assigned,
<u>10</u> | |

3.

- a. From the plot [3a](#) we see important differences between the two histograms. The one for the students from urban areas have a peak at 8, while the histogram for the students from the rural areas have a peak around mark 5. There are large discrepancies between the results from the two group of students.
- b. In the figure [3b](#) the two histograms have a similar shape. The girls' histogram is a bit shifted to higher marks.

Continue the discussion analysing the possible causes for these results.

Exercise 4. *Collisions experiment.*

The typical experiments in particle physics are collisions experiments. To understand better one can simulate them at macroscopic scale. Let's have a fix target in the horizontal plane on which we throw balls in OX direction. Follow the direction of the balls after hitting the target and registrate the deflection angles. The exercise asks to draw the histograms for the two targets in the figure 4, when the beam contains 1000 balls, uniformly distributed on the target cross section.

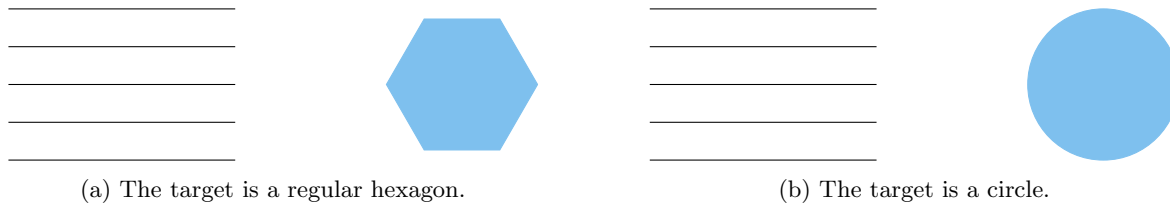


Figure 4: The scattering of a beam of particles on targets with different shapes.

1. The target is a regular hexagon like in the figure 4a. Draw the histogram in the figure ??.
2. The target is a circle in the figure 4b. Draw the histogram in the figure ??.

Solution of the exercise 4.

1. When the target's shape is a hexagon like in the figure, the balls "see" only two faces. One face is tilted 60° with respect to the OX axis and the other is tilted -60° .

The balls deflected by the two faces go in the direction of 120° , respectively -120° , with the OX axis. In the conditions given by the problem, uniformly distributed balls, the histogram 5b is symmetric around the 0 angle and has only two bins of equal sizes⁴.

2. For the case when the target has the shape of a circle 4b, the balls see infinitely many small faces thus there are infinitely many scattering angles. We can solve the problem through induction on n -regular polygons.

First step is to increase the order of the polygon and we study the scattering on target in the shape of octagon, figure 6a. In this case, the balls see four faces 45° , 135° , -45° , respectively -135° . The histogram is symmetric around origin and will have four bins, but with different sizes. The height of the bins is proportional with the projection of the faces on the OY axis.

The general formulae for the case of the n faces polygon are

⁴In a realistic case, the balls are only probabilistic uniform distributed, thus the bins don't have an equal size. They are only on average equal only after a high number of throws

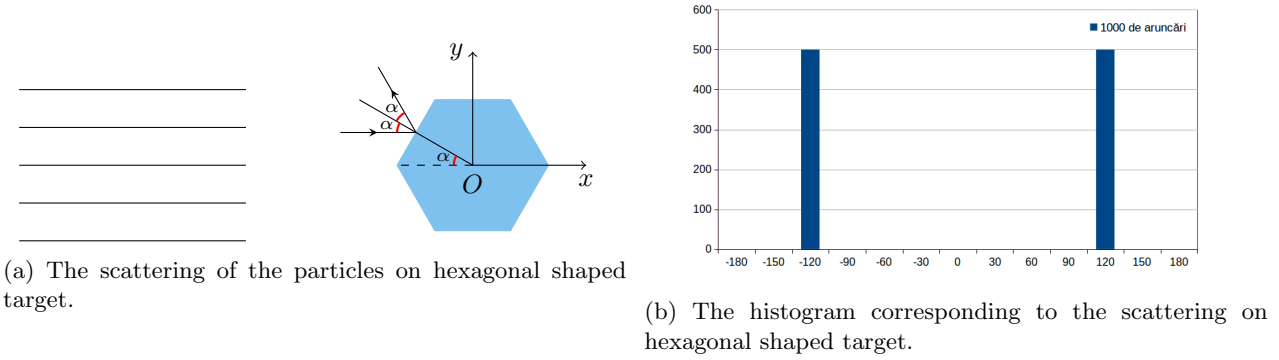


Figure 5: Solution for the hexagon case.

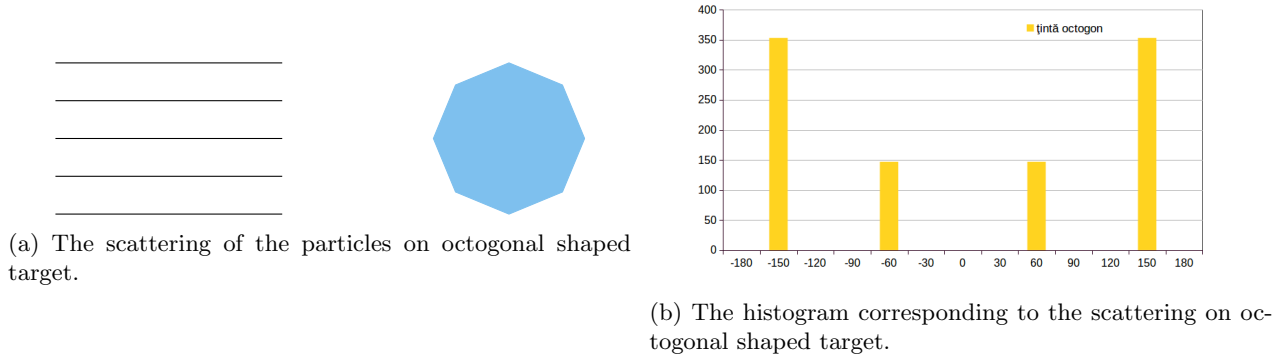


Figure 6: Solution for the octogon case.

$$\alpha = \frac{2\pi}{n}$$

$$x_k = \cos(k\alpha), y_k = \sin(k\alpha) \quad (1)$$

$$s_k = |\sin((k+1)\alpha) - \sin(k\alpha)|$$

$$\beta_k = (\pi - (\pi - \alpha)/2) + k\alpha = \frac{\pi}{2} + \frac{2k+1}{2}\alpha \quad (2)$$

where α is the angle extended by a polygon's face, x_k and y_k are the coordinates (x, y) of the vertex k . The numbering follows the trigonometric direction and starts with the edge closer to OY axis. s_k is the projection on the OY axis of the face between $k+1$ and k vertices and β_k is the direction made by balls deflected on the k -th edge.

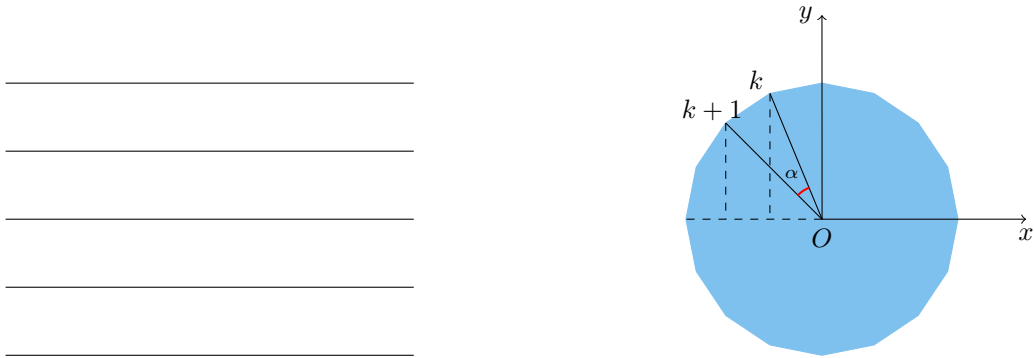
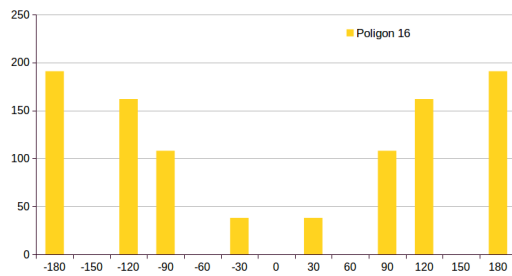
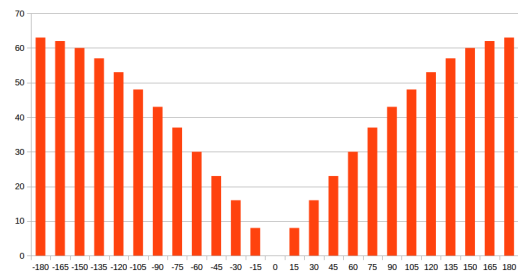


Figure 7: The scattering of the particles on a regular n -polygonal target.



(a) The histogram for the scattering on a regular polygonal with 16 faces.



(b) The histogram for the scattering on a regular polygonal with 50 faces.

Figure 8: The scattering of the particles on regular polygonal targets.



Figure 9: Collision experiment, throwing a projectile ball toward a target linear target made of identical poles distributed non-uniformly on a row of length L .

Exercise 5. *Hands-on activity: indirect measurements*

The following practical activity illustrates the principle of indirect measurements in experimental particle physics.

A number of balls (obstacles) of equal sizes are arranged in a row on a floor see picture 9. The balls simulate the atoms/nuclei from a thin foil of a material and play the role of the target. Their arrangement can be non-uniform but the total length L of the row should be known. It is also known the total number of balls in the target N .

Throw the projectile balls toward the target from a distance of 3 - 5 meters and record if the projectile hit a ball in the target or not. Repeat the experience over 100 times, maintaining the same distance. Let's denote with N_A the total number of throws, and with n_c the number of collisions.

1. Do the experiment in the class, from the values obtained for N_A and n_c compute the radius R of the balls forming the target.
2. Do a direct measurement of the radius of the ball from the target using a ruler and compare it with the value obtained from the indirect measurement.
3. Calculate the absolute error and the relative error of the result in the indirect measurement. How do the errors modify if the experiment is repeated with a projectile ball of a different dimension. de dimensiune diferită.

Indications for the exercise 5.

The activity can be made in different configuration and materials, the most used ones are a) with balls sitting on the floor b) with pieces of wood c) with coins d) drawing the obstacles on a sheet of paper and throwing from above.

Solution

1. Count the number of throws, N_A and the number of collisions n_c . The radius is computed with the formula

$$R = \frac{n_c \cdot L}{2N \cdot N_A} \quad (3)$$

2. To measure more precisely the dimension of the balls put more balls head to head in a line and measure the length of the row and then divide by the number of balls.
3. The absolute error and the relative error are smaller when the projectile radius is smaller. The formula 3 is an approximation which doesn't take into account the dimension of the projectile. The full formula can explain the behaviour of the errors.

Exercise 6. *LHCb Masterclass - measurement the lifetime of D^0 meson*

Follow the instructions from LHCb Masterclass website [1].

References

- [1] LHCb Collaboration. Lhcb international masterclasses. lhcb-public.web.cern.ch/lhcb-public/en/LHCb-outreach/masterclasses/en/.